CAN-EYE Output Variables.

Definitions and theoretical background.

1. INTRODUCTION

Leaf area index indirect measurement techniques are all based on contact frequency (Warren-Wilson, 1959) or gap fraction (Ross, 1981) measurements. Contact frequency is the probability that a beam (or a probe) penetrating inside the canopy will come into contact with a vegetative element. Conversely, gap frequency is the probability that this beam will have no contact with the vegetation elements until it reaches a reference level (generally the ground). The term “gap fraction” is also often used and refers to the integrated value of the gap frequency over a given domain and thus, to the quantity that can be measured, especially using hemispherical images. Therefore, measuring gap fraction is equivalent to measuring transmittance at ground level, in spectral domains where vegetative elements could be assumed black. It is then possible to consider the mono-directional gap fraction which is the fraction of ground observed in a given viewing direction (or in a given incident direction).

The objective of this document is to provide the theoretical background used in the CAN-EYE software to derive canopy biophysical variables from the bi-directional gap fraction measured from the hemispherical images.

2. MODELING THE GAP FRACTION

2.1. LAI definition

The leaf area density, $l(h)$ at level h in the canopy is defined as the leaf area per unit volume of canopy. The leaf area index (LAI) corresponds to the integral of $l(h)$ over canopy height. It is therefore defined as the one sided leaf area per unit horizontal ground surface area (Watson, 1947). Although this definition is clear for flat broad leaves, it may cause problems for needles and non-flat leaves. Based on radiative transfer considerations, Lang (1991) and Chen and Black (1992) and Stenberg (2006) proposed to define *LAI* as half the total developed area of leaves per unit ground horizontal surface area. This definition is therefore valid regardless vegetation element shape.

As defined above, leaf area index, *LAI*, defined as at a level *H* in the canopy is related to the leaf area density through:

Eq. 1 $$LAI = \int_0^H l(h)\, dh$$

2.2. From LAI to Gap Fraction

Following Warren-Wilson (1959), the mean number of contacts $N(H,\theta_v,\varphi_v)$ between a light beam and a vegetation element at a given canopy level *H* in the direction (θ_v,φ_v) is:

Eq. 2 $$N(H,\theta_v,\varphi_v) = \int_o^H G(h,\theta_v,\varphi_v)\, l(h) / \cos\theta_v \, dh$$

where $G(h,\theta_v,\varphi_v)$ is the projection function, i.e. the mean projection of a unit foliage area at level h in direction (θ_v,φ_v). When the leaf area density and the projection function are considered independent of the level h in the canopy, Eq. 2 simplifies in Eq. 3:

Eq. 3 $$N(L,\theta_v,\varphi_v) = G(\theta_v,\varphi_v).LAI / \cos\theta_v$$

The projection function is defined as follows:

Eq. 4 $$\begin{cases} G(\theta_v,\varphi_v) = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}\int_0^{\pi/2} |\cos\psi|\, g(\theta_l,\varphi_l)\sin\theta_l\, d\theta_l\, d\varphi_l & \text{(a)} \\ \cos\psi = \cos\theta_v\cos\theta_l + \sin\theta_v\sin\theta_l\cos(\varphi_v - \varphi_l) & \text{(b)} \end{cases}$$

where $g(\theta_l,\varphi_l)$ is the probability density function that describes leaf orientation distribution function. This induces the two normalization conditions given in Eq. 5a and Eq. 5b.

Eq. 5 $$\begin{cases} \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}\int_0^{\pi/2} g(\theta_l,\varphi_l)\sin\theta_l\, d\theta_l\, d\varphi_l = 1 & \text{(a)} \\ \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}\int_0^{\pi/2} G(\theta_v,\varphi_v)\sin\theta_v\, d\theta_v\, d\varphi_v = \dfrac{1}{2} & \text{(b)} \end{cases}$$

The contact frequency is a very appealing quantity to indirectly estimate *LAI* because no assumptions on leaf spatial distribution, shape, and size are required. Unfortunately, the contact frequency is very difficult to measure in a representative way within canopies. This is the reason why the gap fraction is generally preferred. In the case of a random spatial distribution of infinitely small leaves, the gap fraction $P_0(\theta_v,\varphi_v)$ in direction (θ_v,φ_v) is related to the contact frequency by:

Eq. 6 $$P_0(\theta_v,\varphi_v) = e^{-N(\theta_v,\varphi_v)} = e^{-G(\theta_v,\varphi_v)\cdot LAI/\cos(\theta_v)}$$

This is known as the Poisson model. Conversely to the contact frequency that is linearly related to LAI, the gap fraction is highly non linearly related to LAI. Nilson (1971) demonstrated both from theoretical and empirical evidences that the gap fraction can generally be expressed as an exponential function of the leaf area index even when the random turbid medium assumptions associated to the Poisson model are not satisfied. In case of clumped canopies, a modified expression of the Poisson model can be written:

Eq. 7 $$P_0(\theta_v,\varphi_v) = e^{-\lambda_0 \cdot G(\theta_v,\varphi_v)\cdot LAI/\cos(\theta_v)}$$

where λ_o is the clumping parameter (λ_o <1).

2.3. Modeling the leaf inclination distribution function $g(l,\theta_l,\varphi_l)$

As shown previously, the gap fraction is both related to the leaf area index and the leaf inclination distribution function (LIDF). It is thus necessary to model the leaf inclination distribution function. The azimuthal variation of the LIDF is often assumed uniform and this is the case in the CAN-EYE software, i.e. the probability density function $g(\theta_l,\varphi_l)$ depends only on the leaf normal zenith angle. This assumption is verified in many canopies but may be problematic for heliotropic plants like sunflowers (Andrieu and Sinoquet, 1993).
Among existing models, the ellipsoidal distribution is very convenient and widely used (Campbell, 1986; Campbell, 1990; Wang and Jarvis, 1988): leaf inclination distribution is described by the ratio of the horizontal to the vertical axes of the ellipse that is related to the average leaf inclination angle (ALA variable in CAN-EYE) knowing that $\overline{\theta_l} = \frac{2}{\pi}\int_0^{\pi/2} g(\theta_l)\theta_l d\theta_l$ and that $g(\theta_l)$ is the probability density function that verifies the normalization condition (Eq. 5).

3. ESTIMATING LEAF AREA INDEX AND LEAF INCLINATION FROM GAP FRACTION MEASUREMENTS

3.1. Use of a single direction: LAI57

Considering the inclined point quadrat method, Warren-Wilson (1960) has proposed a formulation of the variation of the contact frequency as a function of the view zenith and foliage inclination angles. Using this formulation, Warren-Wilson (1963) showed that for a view angle of 57.5° the G-function (Eq 4) can be considered as almost independent on leaf inclination (G = 0.5). Using contact frequency at this particular 57.5° angle, Warren-Wilson (1963) derived leaf area index independently from the leaf inclination distribution function within an accuracy of about 7%. Bonhomme et al., (1974) applied this technique using the gap fraction measurements and found a very good agreement between the actual and estimated *LAI* values for young crops.
Therefore, for this particular viewing direction, LAI can be easily deduced from gap fraction:

Eq 8 $$Po(57.5°)=\exp(-0.5LAI/\cos(57.5°)) \Leftrightarrow LAI=\frac{-\ln(Po(57.5°))}{0.93}$$

The CAN-EYE software proposes an estimate of the LAI derived from this equation, called LAI57.

3.2. Use of multiple directions: LAIeff, ALAeff

Among the several methods described in Weiss et al (2004), the LAI estimation in the CAN-EYE software is performed by model inversion since, conversely to the use of the Miller's formula, it can take into account only a part of the zenith angle range sampled by hemispherical images. This is very useful since there is a possibility to reduce the image field of view to less than 90° zenith. This feature is very important due to the high probability of mixed pixels in the part of the image corresponding to large zenith view angles. LAI and ALA are directly retrieved by inverting in CAN_EYE using Eq 6 and assuming an ellipsoidal distribution of the leaf inclination using look-up-table techniques (Knyazikhin et al., 1998; Weiss et al., 2000). A large range of random combinations of *LAI* (between 0 and 10, step of 0.01) and *ALA* (10° and 80°, step of 2°) values is used to build a database made of the corresponding gap fraction values (Eq 6) in the zenithal directions defined by the CAN-EYE user (parameter window definition during the CAN-EYE processing). The process consists then in selecting the LUT element in the database that is the closest to the measured P_o. The distance (cost function C_k) of the k^{th} element of the LUT to the measured gap fraction is computed as the sum of two terms:

Eq. 7. CAN-EYE V5.1 $$J_k = \sqrt{\underbrace{\frac{\sum_{i=1}^{Nb\_Zenith\_Dir} w_i\left(P_o^{LUT(k)}(\theta_i) - P_o^{MES}(\theta_i)\right)^2}{\sigma_{MOD}(P_o^{MES}(\theta_i))}}_{First\quad Term} + \left(\underbrace{\frac{ALA^{LUT(k)} - 60}{30}}_{Second\quad Term}\right)^2}$$

Eq. 8. CAN-EYE V6.1 $$J_k = \sqrt{\underbrace{\frac{\sum_{i=1}^{Nb\_Zenith\_Dir} w_i\left(P_o^{LUT(k)}(\theta_i) - P_o^{MES}(\theta_i)\right)^2}{\sigma_{MOD}(P_o^{MES}(\theta_i))}}_{First\quad Term} + \left(\underbrace{\frac{PAI^{LUT(k)} - PAI^{MES57}}{\sigma_{PAI57}}}_{Second\quad Term}\right)^2}$$

The first term computes a weighted relative root mean square error between the measured gap fraction and the LUT one. The weights w_i take into account the fact that some zenithal directions may contain a lot of masked pixel and therefore, the corresponding gap fraction may not be very representative of the image:

Eq. 8 $$w_i = \frac{NPix_i - Nmask_i}{NPix_i}, \quad \sum_{i=1}^{Nb\_Zenith\_Dir} w_i = 1$$

The relative root mean square error is divided by a "modelled" standard deviation of the measured gap fraction derived from the empirical values $\sigma(P_o^{MES}(\theta_i))$ computed from the images corresponding to the same plot for each zenithal direction *I*, when estimating the measured gap fraction after the CAN-EYE classification step. In order to smooth σ zenithal variations, a second order polynomial is fitted on $\sigma(P_o^{MES}(\theta_i))$ to provide $\sigma_{MOD}(P_o^{MES}(\theta_i))$.

The second term of Eq. 7 is the regularization term (Combal et al, 2002), that imposes constraints on the retrieved ALA values

The LUT gap fraction that provides the minimum value of J_k is then considered as the solution. The corresponding LAI and ALA provide the estimate of the measured CAN-EYE leaf area index and average leaf inclination angle. As there is no assumption about clumping in the expression of the gap fraction used to simulate the LUT (Eq. 6), the foliage is assumed randomly distributed, which is generally not the case in actual canopies. Therefore, retrieval of *LAI* based on the Poisson model and using gap fraction measurements will provide estimates of an effective *LAI*, LAI^{eff}, and corresponding average inclination angle *ALAeff* that allows the description of the observed gap fraction assuming a random spatial distribution.

3.3. From effective leaf are index to true LAI

The "true *LAI*", that can be measured only using a planimeter with however possible allometric relationships to reduce the sampling (Frazer et al., 1997), is related to the effective leaf area index through:

Eq. 8 $$LAI^{eff} = \lambda_o \, LAI$$

where λ_o is the aggregation or dispersion parameter (Nilson 1971; Lemeur and Blad, 1974) or clumping index (Chen and Black, 1992). It depends both on plant structure, *i.e.* the way foliage is located along stems for plants and trunks branches or shoots for trees, and canopy structure, *i.e.* the relative position of the plants in the canopy. The shape and size of leaves might also play an important role on the clumping.

In CAN-EYE, the clumping index is computed using the Lang and Yueqin (1986) logarithm gap fraction averaging method. The principle is based on the assumption that vegetation elements are locally assumed randomly distributed. Each zenithal ring is divided into groups (called cells) of individual pixels. The size of the individual cells must compromise between two criterions: it should be large enough so that the statistics of the gap fraction are meaningful and small enough so that the assumption of randomness of leaf distribution within the cell is valid. For each cell, Po is computed as well as its logarithm. If there is no gap in the cell (only vegetation, i.e, P_o=0), P_o is assumed to be equal to a P_o^{sat} value derived from simple Poisson law, using a prescribed LAI^{sat} value. $P_o^{cell}(\theta)$, as well as $\ln(P_o^{cell}(\theta))$ are then averaged over the azimuth and over the images for each zenithal ring. The averaging still takes into account the masked areas using w_i . The ratio of these two quantities provides the clumping parameter λ_o for each zenithal ring:

$$\lambda_o(\theta, ALA^{eff}) = \frac{mean\left[\log(P_o^{Cell}(\theta))\right]}{\log\left[mean(P_o^{Cell}(\theta))\right]}$$

Note that since P_o^{sat} is simulated using the Poisson model, it depends on the value chosen for both LAI^{sat} and the average leaf inclination angle, the clumping parameter is computed for the whole range of variation of ALA and a LAI^{sat} varying between 8 and 12 (Note that all the results in the CAN-EYE html report are provided for $LAI^{sat} = 10$. Then the same algorithm, as described previously for effective LAI (§3.2), is applied by building a LUT using the modified Poisson model (eq 7) to provide LAI^{true} and ALA^{true} as well as the corresponding clumping parameter.

3.4. LAI or PAI?

Claiming that devices and associated methods based on gap fraction measurements provide an estimate of the leaf area index is not right since indirect measurements only allow assessing plant area index. Indeed, it is not possible to know if some leaves are present behind the stems, branches or trunk. Therefore, masking some parts of the plants (which is possible using CAN-EYE) to keep only the visible leaves is not correct and could lead to large under-estimation of the actual LAI value, depending on the way leaves are grouped with the other parts of the plant. Therefore, all CAN-EYE outputs correspond to plant area index and not leaf area index.

4. COMPUTATION OF THE COVER FRACTION

Cover fraction (fCover) is defined as the fraction of the soil covered by the vegetation viewed in the nadir direction:

Eq 9. $$fCover = 1 - P_o(0)$$

Using hemispherical images, it is not possible to get a value in the exact nadir direction, and the cover fraction must be integrated over a range of zenith angles. In CAN-EYE, the default value for this range is set to 0-10°. The user can change this value when defining the CAN-EYE parameters (which also concerns the description of the hemispherical lens properties) at the beginning of the processing.

5. FAPAR COMPUTATION

fAPAR is the fraction of absorbed photosynthetically active radiation (400-700nm) by the vegetation. It varies with sun position. As there is little scattering by leaves in that particular spectral domain due to the strong absorbing features of the photosynthetic pigments {Andrieu, 1993 #10), fAPAR is often assumed to be equal to fIPAR (fraction of Intercepted photosynthetically active radiation), and therefore to the gap fraction. The actual fAPAR is the sum of two terms, weighted by the diffuse fraction in the PAR domain: the 'black sky' fAPAR that corresponds to the direct component (collimated beam irradiance in the sun direction only) and the 'white sky' or the diffuse component. The closest approximation to white sky fAPAR occurs under a deep cloud cover that may generate an almost isotropic diffuse downward. Following Martonchik et al {, 2000 #578}, the adjectives black and white are not related to the color of the sky, but rather to the angular distribution of light intensity.
Providing the latitude and the date of the image acquisition, the CAN-EYE software proposes three outputs for fAPAR:

1- The instantaneous 'black sky' fAPAR ($fAPAR^{BS}$): it is the black sky fAPAR at a given solar position (date, hour and latitude). Depending on latitude, CAN-EYE computes the solar zenith angle every solar hour during half the day (there is symmetry at 12:00). The instantaneous fAPAR is then approximated at each solar hour as the gap fraction in the corresponding solar zenith angle:

$$fAPAR^{BS}(\theta_s) = 1 - P_o(\theta_s)$$

2- The daily integrated black sky (or direct) fAPAR is computed as the following::

$$fAPAR^{BS}_{Day} = \frac{\int_{sunset}^{sunrise} \cos(\theta)(1 - P_o(\theta))d\theta}{\int_{sunset}^{sunrise} \cos(\theta)d\theta}$$

3- The white sky (or diffuse) fAPAR is computed as the following:

$$fAPAR^{WS} = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\pi/2}(1 - P_o(\theta))\cos\theta\sin\theta d\theta d\varphi = 2\int_0^{\pi/2}(1 - P_o(\theta))\cos\theta\sin\theta d\theta$$

6. SUMMARY OF ESTIMATED VARIABLES

Variable	Acronym	Paragraph
Effective Leaf Area Index estimated from $P_o(57°)$	LAI57	3.1
Effective Leaf area index	LAIeff	3.2
Effective average leaf inclination angle	ALAeff	3.2
True leaf area index	LAItrue	3.3
True average leaf inclination angle	ALA true	3.3
Clumping Factor	CF	3.3
Cover Fraction	fCover	4
Instantaneous 'black sky'fAPAR	$FAPAR^{BS}$	5
White sky fAPAR	$FAPAR^{WS}$	5

Daily black Sky fAPAR	$fAPAR^{BS}_{Day}$	5

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